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ON THE STRIBECK's NUMBERS IN RADIALLY LOADED ROLLING ELEMENT BEARINGS

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Resumo*. Stribeck investigou a distribuição de cargas nos elementos rolantes de um rolamento de esferas carregado radialmente e descobriu que a carga máxima da esfera poderia ser obtida multiplicando a carga média por 4,37, para folga interna zero. Esse valor passou a ser conhecido como Constante ou Número de Stribeck e é utilizado até hoje para dimensionamento de rolamentos. Mais tarde, Palmgren afirmou que, para rolamentos de rolos com folga interna zero, o Número de Stribeck é 4,08. Esse trabalho mostra como foram obtidas as Constantes de Stribeck e que são aproximações do limite da relação entre a carga no elemento mais carregado e a carga média, quando o número de elementos rolantes no mancal tende para o infinito. Também é mostrado que o erro ao adotar o valor 4,08 para a Constante de Stribeck, que representa o número que deve ser multiplicado pela carga radial externa média para obter a carga radial máxima do rolo, é 55,6 vezes maior que ao adotar o valor 4,37 para a Constante de Stribeck, que representa o número que deve ser multiplicado pela carga radial externa média para obter a carga radial máxima da esfera.*

Palavras chave*: rolamento, número de Stribeck, carga radial.*

Abstract. *Stribeck investigated the load's distribution on the rolling elements of a radially loaded ball bearing and found that the maximum ball load could be found multiplying the medium load by 4.37, for zero internal clearance. This number came to be known as Stribeck's Constant or Number and it's still used today for bearing sizing. Later, Palmgren stated that the theoretical value of Stribeck's Constant for roller bearings with zero internal clearance is 4.08. This work shows how the Stribeck's Constants were found, which are approximations of the limits of the ratios between the load on the most heavily loaded element and the medium load, when the number of rolling elements in the bearing tends to infinity. It's also shown that the error when adopting the value 4.08 for the Stribeck's Constant, which represents the number that must be multiplied by the medium external radial load to obtain the maximum radial roller load, is 55.6 times greater than when adopting the value 4.37 for the Stribeck's Constant, which represents the number that must be multiplied by the medium external radial load to obtain the maximum radial ball load.*

Keywords: *rolling element bearing, Stribeck's number, radial load.*

1. INTRODUCTION

Many machines and mechanical devices have shafts supported by rolling element bearings. A rolling element bearing provides load transmission between two structures, generally called a *shaft* and a *housing*, maintaining the relative position between the structures, but allowing 1 or two rotational and translational degrees of freedom. The stiffness of the shaft in relation to the housing, as well as the device's performance and life are strongly influenced by internal parameters and external loads. The external load of the bearing is transferred from one ring to the other through the rolling elements, and the stiffness depends on the distribution of the load across the rolling elements, which is not uniform. The degree of uneven load distribution depends on the internal geometry of the bearing and the magnitude of the external load. An internal parameter that has a strong influence is the diametral clearance.

Stribeck (1907) investigated the distribution of loads on the rolling elements of a radially loaded ball bearing and found that the *maximum* ball load could be obtained by multiplying the *medium* load by 4.37, for zero internal clearance. This number came to be known as Stribeck's Constant or Number and, to account for nonzero diametrical clearance and other effects, Stribeck recommended rounding the Constant to 5.0.

Palmgren (1959) stated that the theoretical value of Stribeck's Constant for roller bearings with zero internal clearance is 4.08 and suggested using Stribeck's recommended value of 5.0 for the Stribeck's Constant for either ball or roller bearings having typical clearance.

Mitrović (2001) investigated how internal radial clearance influences on the static load rating of the deep groove ball bearing. This effort included a comparative analysis of the static load rating determined under the ISO recommendations, on the one side, and the real load rating, on the other, from the standpoint of the influence of internal radial clearance.

Lazović *et al*, 2008, showed that real load distribution in radially loaded ball bearing is between two boundary load distributions: ideally equal and extremely unequal loads distributions. They developed a mathematical model of load distribution respecting classic rolling bearing theory and they had introduced, a new, original parameter defined as load distribution factor. Developed mathematical model includes all main influences on load distribution in rolling bearing (number of rolling elements, internal radial clearance and external load).

Research has shown that ball bearings should have slightly negative or zero internal clearance to maximize bearing life and reliability. However, zero or slightly negative clearance generates high contact stresses between raceways and balls (Oswald *et al*., 2012; Xiaoli *et al*., 2017). In addition, it increases bearing friction, which makes the bearing vulnerable to temperature rise. Therefore, a bearing with positive diametral clearance is generally chosen to avoid such problems (Sinha and Sahoo, 2020).

Oswald *et al*., 2012, investigated the effect of internal clearance on load distribution and fatigue life of radially loaded deep groove ball bearings. They found that life gradually decreases with increasing clearance's absolute value and is maximum under small negative clearance. Furthermore, in radially loaded bearings, the rolling elements, which transfer the load – active elements –, are located below the meridian plane in the so-called *loading zone*. The quasi-static analysis to derive the radial load distribution is based on the assumption that the inner ring, under load, moves radially in the direction of the external load, with respect to the outer ring, which rings are considered rigid.

Xiaoli *et al*., 2017, developed a mathematical model based on the Hertz elastic contact theory to determine the radial load distribution in ball bearings with positive, negative and zero clearance.

Zhang *et al*., 2017, proposed a model to study the load distribution in a ball bearing as a preload's function. Preload means negative radial internal clearance. The new model is then used to calculate fatigue life as a function of external load, rotational speed and preload. The results show that adequate preload improves load distribution and prolongs fatigue life. Given load and rotational speed, an optimal preload can be obtained, estimated through the life of the bearing.

Using the Lundberg and Palmgren model and the finite element method, Chudzik and Warda (2019) determined the stress distributions needed to estimate the fatigue life of a radial cylindrical roller bearing under the influence of radial internal clearance. The result reveals that the radial cylindrical roller bearing will have higher fatigue strength with a slight radial clearance.

Considering two different distributions of the balls in relation to the load line, Sinha and Sahoo (2020) obtained a better load distribution in ball bearings under the influence of the relative motion between the rings in the presence of radial clearance.

Ball bearing clearance influences starting torque, bearing stiffness and affects load distribution, load carrying capacity and bearing life. Sinha *et al*., 2021, calculated the radial load distribution in a deep groove ball bearing with variable clearance - which is obtained by constructing the outer ring's groove in an elliptical shape - and the results shown that the load distribution is better when compared to the conventional bearing, for $|\theta| < \pi/4$, where θ is the angle between the load line and the major axis of the elliptical outer groove.

Ricci (2009a, 2009b) has developed numerical procedures whose analogues haven't been found in the literature. The equilibrium solutions for internal load distribution in single row, statically loaded angular contact ball bearings subject to known radial, axial and combined moment's loads, are derived using the Newton-Rhapson method. The procedure has been used to find load distribution differences between a loaded, unfitted bearing at room temperature and the same bearing loaded with interference fits, which can experience radial temperature gradients between the inner and outer rings. For each step of the procedure, the iterative solution of $z + k$ simultaneous algebraic nonlinear equations is required - where z is the number of balls and k is a small positive integer - to produce an exact solution for the axial and radial deflections and the contact angles.

Ricci (2022) expanded the work of Lazović *et al* to account for clearance sign in radially loaded rolling element bearings and also included results for cylindrical roller bearings.

The literature shows that the internal parameters, and nature and type of external load influence the load distribution, load capacity, stiffness, dynamical performance, and life of bearings. In this work, we dedicate ourselves to studying a little more how the load distribution between the rolling elements occurs, for a bearing with zero diametral clearance. For this we proceed to revisit Stribeck's pioneering work.

We show here that the well-known Stribeck's constants, used still today to sizing bearings, are ratio's values approximations between the load on the most heavily loaded element and the average load, when the number of rolling elements tends to infinity. We calculated the error, in relation to the value obtained numerically, when assuming the Stribeck's Constants as the industry standard for this ratio, as well as showing that the error when adopting 4.08 for the Constant related to roller bearings is much greater than that made when 4.37 is adopted for the ball bearing Constant.

2. RATIO BETWEEN THE RADIAL EXTERNAL LOAD AND THE MAXIMUM ROLLING ELEMENT's LOAD

Figure 1, taken from Stribeck (1907), translated from the German, shows the distribution of an external radial load, *P*, on the rolling elements of a radial bearing with one row of rolling elements. The external radial load line, *W*0, passes through the center of the rolling element number zero, and is distributed to the other rolling elements 1, 2, 3, etc. The rolling elements are evenly spaced angularly by a cage. Therefore, the angular distance between two rolling elements is a constant given by $\gamma = 360°/z$, where *z* is the number of rolling elements in a single row rolling element bearing.

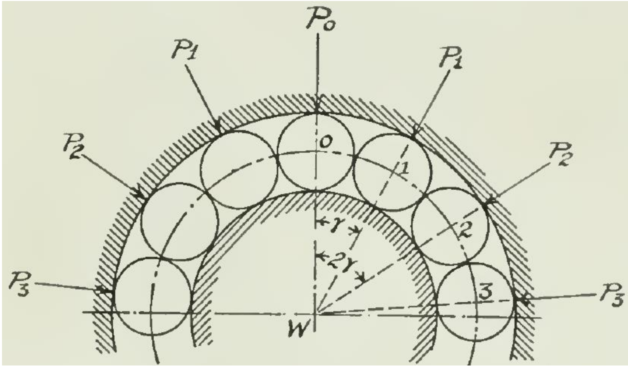


Figure 1. Distribution of a radial external load, *P*, on the rolling elements of a radial bearing
Source: Stribeck (1907), p. 451

To self-content with Stribeck's description, for a bearing with a null radial clearance, if the rings are not appreciably deformed under the P_j loads, then

$$\delta_j = \delta_0 \cos(j\gamma), \qquad j = 1, ..., n, \tag{1}$$

where δ_0 is the approximation of the two rings in the direction of the radius *W*0 under load P_0, δ_j are the deformations corresponding to P_j, and *n* is the greater integer less than $90°/\gamma = z/4$, i.e., the *pairs* number of rolling elements that effectively work in the load transfer, beyond the ball "0".

Considering the distribution shown in Fig. 1, yields

$$P = P_0 + 2\sum_{j=1}^{n} P_j \cos(j\gamma). \tag{2}$$

Since, for *ball bearings* (Stribeck, 1907),

$$P_0 = k_b \delta_0^{3/2}, \quad P_j = k_b \delta_j^{3/2}, \qquad j = 1, ..., n, \tag{3}$$

where k_b is a load-deflection proportionality constant, then

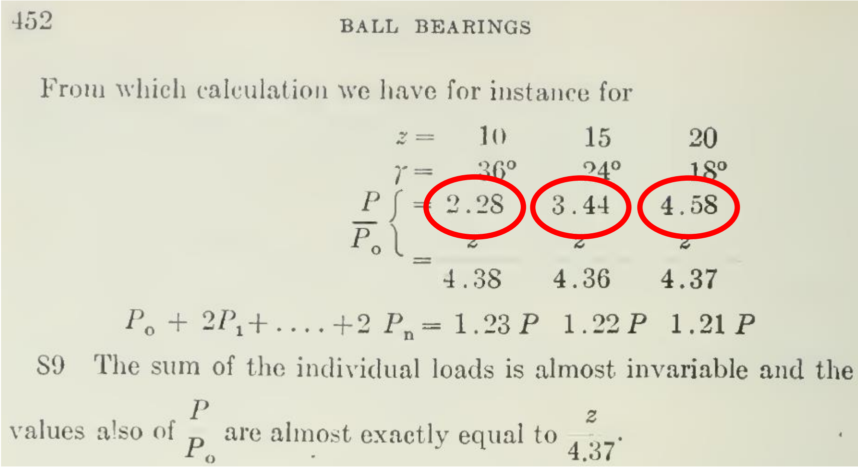
$$\frac{P}{P_0} = 1 + 2\sum_{j=1}^{n} \cos^{5/2}(j\gamma). \tag{4}$$

Similarly, for *roller bearings* (Palmgren, 1959),

$$P_0 = k_r \delta_0^{10/9}, \quad P_j = k_r \delta_j^{10/9}, \qquad j = 1, ..., n, \tag{5}$$

$$\frac{P}{P_0} = 1 + 2\sum_{j=1}^{n} \cos^{19/9}(j\gamma). \tag{6}$$

Figure 2 shows the excerpt taken from Stribeck's paper, where he comes to the conclusion that the constant, which must be multiplied by the medium load to obtain the maximum ball load, in a radially loaded ball bearing, is 4.37. The values circled by the red ellipses are values obtained by Stribeck from the P/P_0 ratio for bearings with 10, 15 and 20 balls.



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From which calculation we have for instance for

$z =$	10	15	20
$\gamma =$	36°	24°	18°
$\frac{P}{P_0} =$	2.28	3.44	4.58
	$\frac{z}{4.38}$	$\frac{z}{4.36}$	$\frac{z}{4.37}$
$P_0 + 2P_1 + \dots + 2P_n =$	$1.23\,P$	$1.22\,P$	$1.21\,P$

89 The sum of the individual loads is almost invariable and the values also of $\frac{P}{P_0}$ are almost exactly equal to $\frac{z}{4.37}$.

Figure 2. Excerpt taken from Stribeck's paper, where he comes to the conclusion that the constant, which must be multiplied by the medium load to obtain the maximum ball load, is 4.37
Source: Stribeck (1907), p. 452

Figure 3 shows the plots of the ratios between the radial external load, P, and the maximum rolling element's radial load, P_0, that is, the plots of Eqs. (4) and (6), using a numerical algorithm developed by this author, as a function of the total number z of rolling elements in the bearing. In these plots, z ranges from 2 to 20. The blue line is for ball bearing and the red line is for cylindrical roller bearing. The number of rolling elements (balls and rollers) that work to transfer the radial load, $2n + 1$, is also shown in Fig. 3. The left-hand ordinate gives the P/P_0 ratio values for ball and roller bearings and the right-hand ordinate gives the effective number of rolling elements.

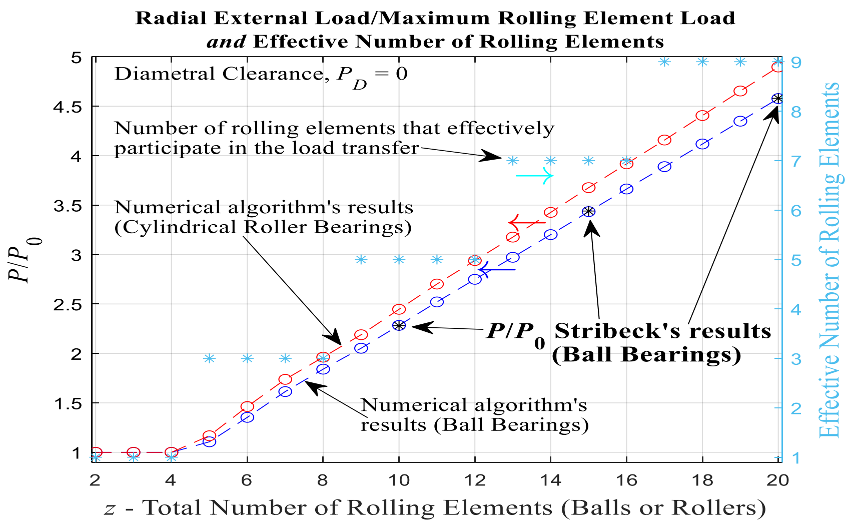


Figure 3. Ratio between radial external load, P, and maximum rolling element's load, P_0; and effective number of rolling elements, as z's functions

Interestingly, for the configuration shown in Fig. 1, where the line of the external load passes through the most loaded rolling element's center, the sum of the elements that work on transferring the external load is always odd. It is the sum of the most loaded rolling element plus n pairs of rolling elements, where n times γ never exceeding 90º. Remember that the rings are considered rigid and any deflection of the one ring relative to the other always will deform the $2n + 1$ elements, since the diametral clearance is considered to be zero. Also note that the values adopted by Stribeck for 10, 15 and 20 balls, circled by the red ellipses in Fig. 2, are exactly on the straight line obtained by the numerical algorithm, based on Eq. (4).

3. RATIOS BETWEEN THE MAXIMUM ROLLING ELEMENT's LOAD AND THE MEDIUM RADIAL EXTERNAL LOAD (STRIBECK's COEFFICIENTS FOR ROLLING ELEMENTS BEARINGS)

Equations (4) and (6) can be inverted and multiplied by z to obtain, for *ball bearings*,

$$\frac{zP_0}{P} = \frac{z}{1 + 2\sum_{j=1}^{n} \cos^{5/2}(j\gamma)} \tag{7}$$

and for *roller bearings*,

$$\frac{zP_0}{P} = \frac{z}{1 + 2\sum_{j=1}^{n} \cos^{19/9}(j\gamma)}. \tag{8}$$

Equations (7) and (8) represent the ratios between the maximum rolling element's radial load, P_0, and the medium radial external load, P/z, for ball bearings and cylindrical roller bearings, respectively. These ratios are defined as Stribeck's Coefficients, S_t, for a ball and roller bearings and differ from Stribeck's Constants or Stribeck's Numbers, which are approximations for Stribeck's Coefficients.

Figure 4 shows the plots of Eqs. (7) and (8) as z's functions, obtained using a numerical algorithm developed by author. In these plots z ranges from 2 to 20. The left-hand ordinate in Fig. 4 gives the zP_0/P ratio values for ball bearings (blue line) and for cylindrical roller bearings (red line). It's possible to see clearly that, with increasing z, the zP_0/P ratio tends to a certain value close to 4.37 for ball bearings and 4.08 for roller bearings.

Figure 4 also shows the plots of the Stribeck's Constants' error values as compared to the Stribeck's Coefficients obtained numerically. The right-hand ordinate gives the relative Stribeck's Constants' error values.

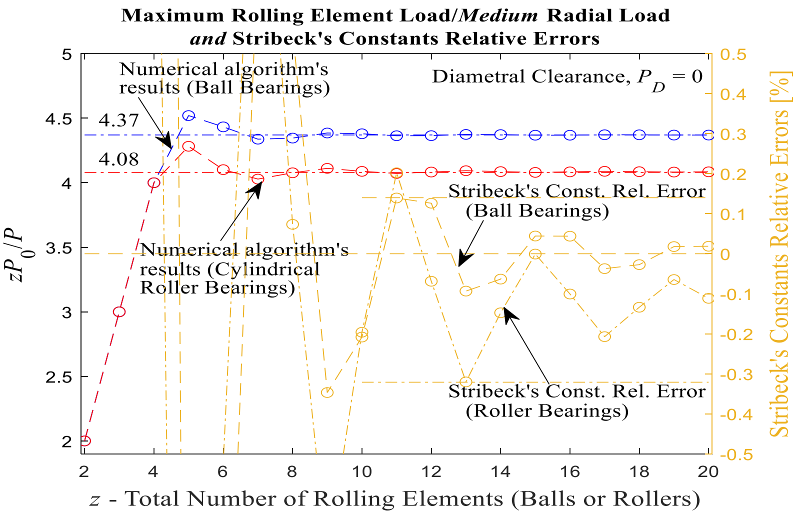
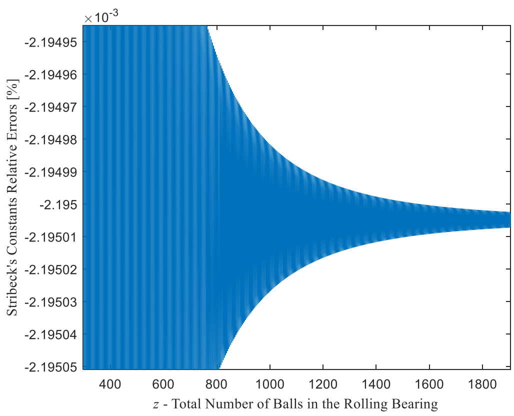
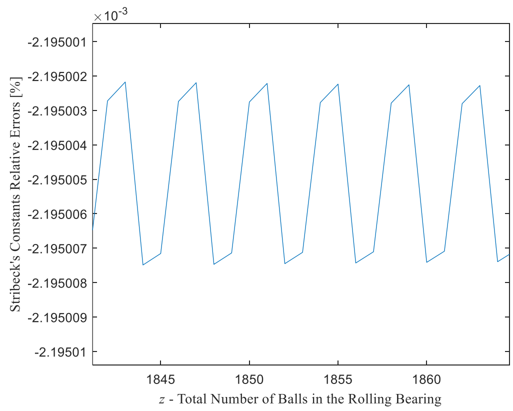


Figure 4. Ratio between maximum rolling element's load and medium radial external load; and Stribeck's Constants relative errors, as z's functions

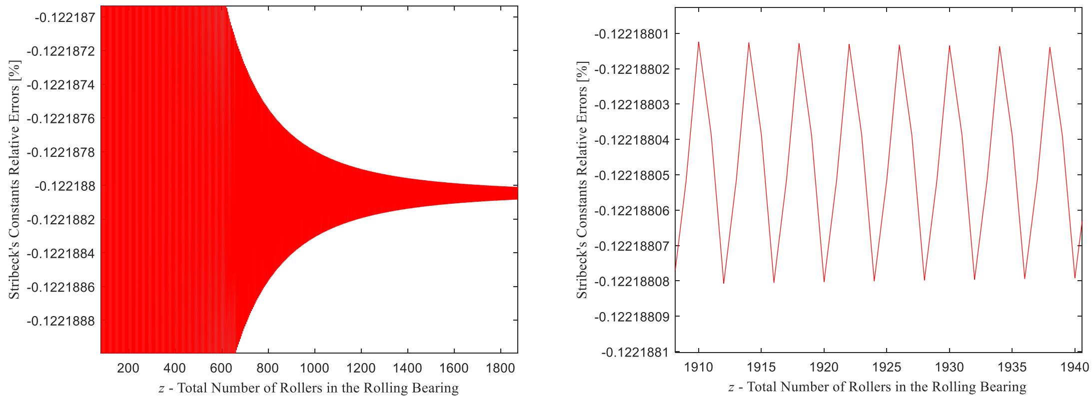
For $z \to \infty$ the relative numerical error tends to an irrational number, oscillating with amplitudes that decay exponentially. For ball bearing the error has a square waveform and tends to $-0.002195...\%$ (Fig. 5); and for roller bearing a triangular waveform, tending to $-0.122188...\%$ (Fig. 6).





(*a*) (*b*)

Figure 5. *a*) Relative error of the Stribeck's Constant for ball bearings, as *z*'s functions; *b*) *detail*



(*a*) (*b*)

Figure 6. *a*) Relative error of the Stribeck's Constant for cylindrical roller bearings, as *z*'s functions; *b*) *detail*

Once the error decreases with *z*, and Stribeck has chosen the constant 4.37 considering bearings with 10, 15 and 20 balls, the relative error in this range is already very small, less than 0.14%, when compared to smaller *z* values. The relative error in the range of 10 to 20 cylindrical rollers is already also small, less than 0.33% (see Fig. 4).

Table 1 shows in table form the data already shown in Figs. 3 and 4. A similar table can be found in the Oswald *et al*., 2012, with some slightly different values. However, this small disparity in tabulated values was not enough to change the conclusion reached by Oswald *et al*., namely, that the Stribeck's Constants relative errors are lesser than 1%, if there are at least seven balls or eight rollers in the bearing. The 4.08 value is 6.6% lower than the Stribeck's Constant for ball bearings.

Furthermore, by increasing the *z* values, it is observed that the relative error between the value adopted for the Stribeck's Constant, in relation to the numerical approach, tends approximately to -2.195×10^{-3}%, for ball bearings, and approximately to –0.122188%, for roller bearings. Therefore, the error when adopting the value 4.08 for the Stribeck's Constant, which represents the number that must be multiplied by the medium external radial load to obtain the maximum radial roller load, is 55.6 times greater than when adopting the value 4.37 for the Stribeck's Constant, which represents the number that must be multiplied by the medium external radial load to obtain the maximum radial ball load.

Table 1. Stribeck's Coefficients, zP_0/P, for a rolling element bearings with zero diametral clearance

Number of rolling elements, *z*	Angle between rolling elements, γ [Deg]	Effective number of rolling elements, $2n+1$	Radial Ball Bearings			Cylindrical Roller Bearings		
			P_0/P	zP_0/P	Relative Error for 4.37, [%]	P_0/P	zP_0/P	Relative Error for 4.08, [%]
2	180	1	1.000	2.000	118.5	1.000	2.000	104.0
3	120	1	1.000	3.000	45.67	1.000	3.000	36.00
4	90	1	1.000	4.000	9.25	1.000	4.000	20.00
5	72	3	1.106	4.520	-3.32	1.168	4.282	-4.72
6	60	3	1.354	4.433	-1.4	1.463	4.101	-0.52
7	51.43	3	1.614	4.337	0.75	1.738	4.028	1.28
8	45	3	1.841	4.346	0.56	1.962	4.077	0.07
9	40	5	2.052	4.385	-0.35	2.189	4.111	-0.76
10	36	5	2.284	4.379	-0.21	2.446	4.088	-0.20
15	24	7	3.434	4.368	0.04	3.676	4.080	-0.00
20	18	9	4.578	4.369	0.02	4.896	4.085	-0.11
30	12	15	6.865	4.370	-0.01	7.344	4.085	-0.124
50	7.2	25	11.44	4.370	-0.00	12.24	4.085	-0.123

4. RATIOS BETWEEN THE *J*-TH LOADED ROLLING ELEMENT's LOAD AND THE MEDIUM RADIAL EXTERNAL LOAD

Whereas for *ball bearings*,

$$P_0 = P_j \cos^{-3/2}(j\gamma), \tag{9}$$

then, substituting Eq. (9) in Eq. (7), yields

$$\frac{zP_j}{P} = \frac{z\cos^{3/2}(j\gamma)}{1+2\sum_{j=1}^{n}\cos^{5/2}(j\gamma)}. \tag{10}$$

Similarly, for *roller bearings*,

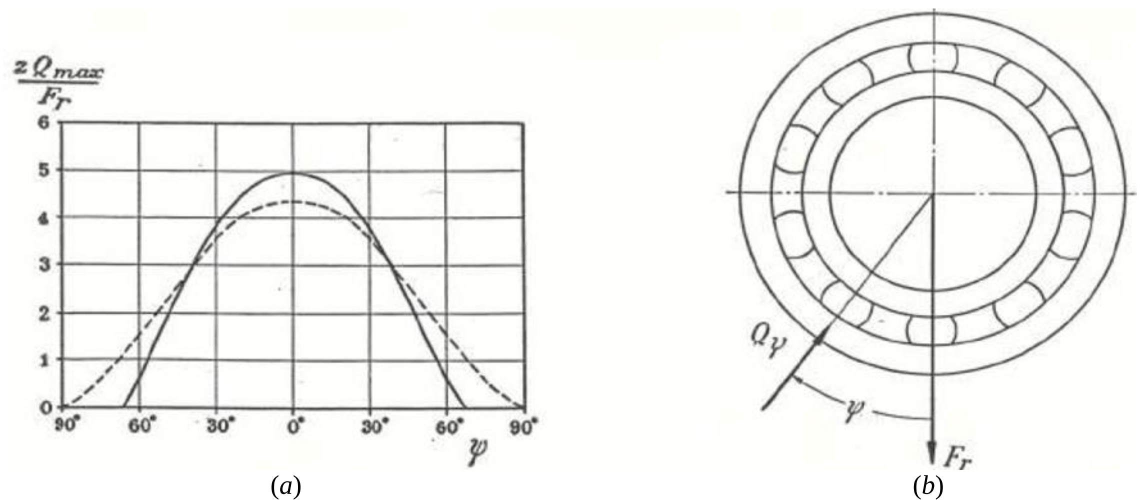
$$P_0 = P_j \cos^{-10/9}(j\gamma), \tag{11}$$

which substituted in Eq. (8), yields

$$\frac{zP_j}{P} = \frac{z\cos^{10/9}(j\gamma)}{1+2\sum_{j=1}^{n}\cos^{19/9}(j\gamma)}. \tag{12}$$

Equations (10) and (12), represent the ratios between the *j*-th loaded rolling element radial load and the medium radial external load, for ball bearings and for cylindrical roller bearings, respectively.

Figure 7 is taken from the Arvid Palmgren's book, who stated that experimental research, subsequent to Stribeck's paper, confirmed the results of Stribeck's Constants for zero and standard clearances, 4.37 and 5, respectively; and he showed experimental results for the load distribution on the balls, measured in a self-aligning ball bearing No. 1309, having a radial internal clearance of 0.01 mm, a clearance of 0.05 mm between the outer ring and the bearing housing under an external radial load of 675 kg. Such a distribution is shown by the solid line in Fig. 7(*a*), where the ordinate is in the unit zQ_{max}/F_r, so it is directly comparable with the formulas for the Stribeck's Coefficients, zP_0/P. Here, there is a small spelling error, as the unit should be zQ_j/F_r, since, as it's written the result is a constant and would not vary with the abscissa. The abscissa ψ – variable related with γ in Fig. 1 - represents the angle between the annular location of the ball in question and the direction of external radial load (Fig. 7(*b*)). Also, according to Palmgren, the dotted line in Fig. 7(*a*) provides, by way of comparison, the calculated load distribution for the ideal case of zero internal clearance and rigid rings.



(*a*) (*b*)

Figure 7. *a*) Palmgrens's experimental results for the load distribution in a ball bearing, in the unit of Stribeck's Constant, as a function of azimuth angle. *Solid line* is for actual case with a clearance. *Dotted line* is for ideal case of zero internal clearance and rigid rings; (*b*) Azimuth angle for a bearing's rolling element
Source: Palmgren (1959), p. 54

Figure 8 shows the plots of Eq. (10), for *z* equal to 10, 15 and 20; which are plots of the ratios between the radial ball loads, P_j, and the medium external radial load, P/z, versus the azimuth angle ranging from -90º to 90º. These values for *z* were chosen because they are in the *z* range between 10 and 20, a range whose ratio between the maximum ball load and the medium external radial load can be approximated by the Stribeck's Constant, 4.37, with a very small error

- less than 0.14% - in relation to the values obtained numerically. The ratios between the ball loads, which contribute to radial load transfer, and the medium load are indicated by small circles. Polynomial interpolation is used to obtain four lines, three of which related to each *z* value. Experimental values taken from Fig. 7(*a*) for zero radial internal clearance are also shown. Note that the four lines - the three related to *z* values equal to 10, 15 and 20, and one related to the experimental values - are very close, indicating a small error one in relation to the other, and all lines approximate very well the ratios between the ball loads and the medium radial load.

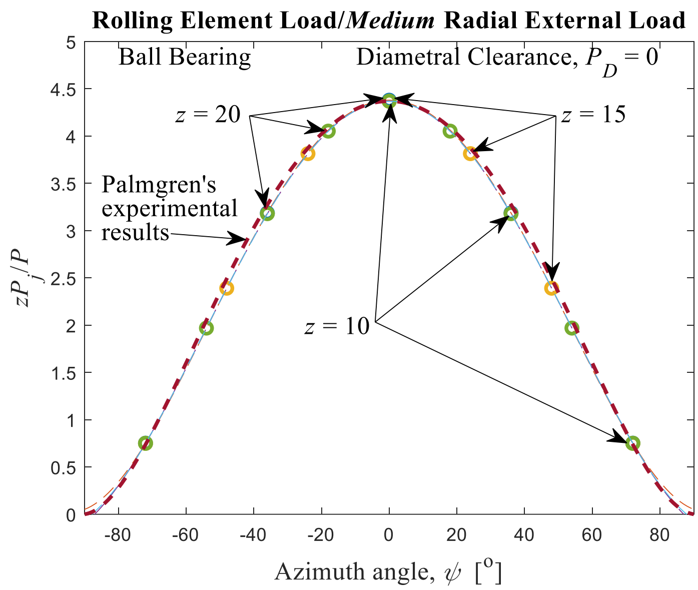


Figure 8. Plots of the ratios between the loads on the balls, P_j, and the medium external radial load, P/z, versus the azimuth angle ranging from -90º to 90º, for *z* equal to 10, 15 and 20; and experimental values - taken from Fig. 7(*a*) - for zero radial internal clearance

Figure 9 shows the plots of equation (12), for *z* equal to 10, 15 and 20; which are plots of the ratios between the roller loads, P_j, and the medium external radial load, P/z, versus the azimuth angle ranging from -90º to 90º. These values for *z* were chosen because they are in the *z* range between 10 and 20, a range whose ratio between the maximum roller load and the medium external radial load can be approximated by the Stribeck's Constant, 4.08, with a very small error - less than 0.33% - in relation to the values obtained numerically. The ratios between the roller loads, which contribute to radial load transfer, and the medium load are indicated by small circles. Polynomial interpolation is used to obtain three lines, each one related to one *z* value. Note that the three lines are very close, indicating a small error of one in relation to the other, and all lines approximate very well the ratios between the ball loads and the medium radial load.

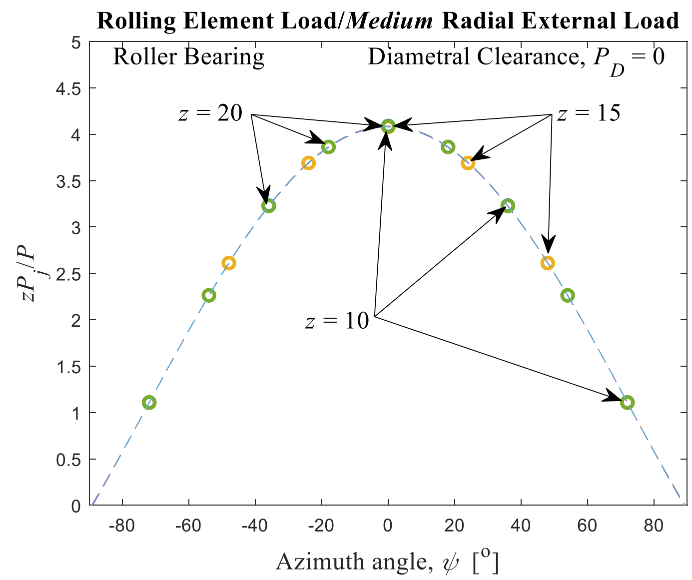


Figure 9. Plots of the ratios between the loads on the rollers, P_j, and the medium external radial load, P/z, versus the azimuth angle ranging from -90º to 90º, for z equal to 10, 15 and 20

5. CONCLUSIONS

Stiffness or performance, as well as load capacity and bearing life are intrinsically linked to the nature and magnitude of external loading, material's properties and internal parameters of rolling element bearings. Contact geometry, load distribution on rolling elements and radial clearance are the internal parameters that most affect bearing performance. In this work, emphasis was given to the distribution of external radial load on rolling elements (balls or cylindrical rollers) of a radially loaded rolling element bearing. Stribeck's pioneering work has been revisited to show that Stribeck's famous Constants are approximations of the ratio of load on the most heavily loaded rolling element to the average load, as the number of bearing elements tends to infinity. Stribeck's Constants were compared with Stribeck's Coefficients' values obtained numerically and errors in the approximations were established. A load distribution of the ratio between the rolling element loads, P_j, to medium load, P/z, as an azimuth angle's function, for ball and roller bearing, were established numerically and the results for ball bearings are close to the experimental values measured by Palmgren.

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8. RESPONSIBILITY FOR INFORMATION

The author is solely responsible for the information included in this work.